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# Resolving the virial discrepancy in clusters of galaxies with modified Newtonian dynamics

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**Abstract.** A sample of 197 X-ray emitting clusters of galaxies is considered in the context of Milgrom's modified Newtonian dynamics (MOND). It is shown that the gas mass, extrapolated via an assumed  $\beta$  model to a fixed radius of 3 Mpc, is correlated with the gas temperature as predicted by MOND ( $M_g \propto T^2$ ). The observed temperatures are generally consistent with the inferred mass of hot gas; no substantial quantity of additional unseen matter is required in the context of MOND. However, modified dynamics cannot resolve the strong lensing discrepancy in those clusters where this phenomenon occurs. The prediction is that additional baryonic matter may be detected in the central regions of rich clusters.

**Key words:** X-rays: clusters of galaxies– dark matter

## 1. Introduction

The discrepancy between the luminous mass and the classical dynamical mass was first identified in clusters of galaxies, the largest virialized systems in the Universe (Zwicky 1933). This discrepancy has been only partially alleviated by the subsequent detection of a substantial component of hot X-ray emitting intracluster gas– a component with a total mass which may, in rich clusters, exceed the stellar mass in galaxies by a factor of four or five (Jones & Forman 1984, David et al. 1990, Böhringer et al. 1993). Even considering the contribution of this diffuse gas, the mass of detectable matter falls by at least a factor of three– more typically a factor of 10– to account for the Newtonian dynamical mass of clusters. The traditional solution to this problem is to postulate the presence of unseen matter which is most often assumed to be non-dissipative and non-baryonic.

Another solution lies in the possibility that there is no substantial quantity of dark matter but that Newtonian gravity or dynamics is not valid on the scale of large astronomical systems. Of several suggested alternatives to dark matter on extragalactic scales, Milgrom's phenomenologically motivated modified Newtonian dynamics (MOND) is the most successful in accounting for the sys-

tematics and details of the discrepancy in galaxies. The basic idea is that, below a critical acceleration,  $a_0 \approx 10^{-8}$  cm/s<sup>2</sup>, the magnitude of the gravitational acceleration is given by  $g = \sqrt{g_n a_0}$  where  $g_n$  is the Newtonian gravitational acceleration (Milgrom 1983a). At higher accelerations,  $g = g_n$  as usual. The low acceleration limit directly implies the luminosity-velocity correlations observed for galaxies ( $L \propto v^4$ )– the Tully-Fisher relation for spirals and the Faber-Jackson relation for ellipticals– as well as predicting that galaxy rotation curves become asymptotically flat in the limit of large distance from the visible galaxy (Milgrom 1983b).

The success of the simple MOND prescription in predicting the detailed shape of the rotation curves of spiral galaxies from the observed distribution of detectable matter is well-documented (Begeman et al. 1991, Sanders 1996, McGaugh & de Blok 1998a, b, Sanders & Verheijen 1998). But MOND, as a modification of Newtonian gravity or inertia, must also account for the observed properties of larger virialized systems which lie in the low-acceleration regime– groups and clusters of galaxies. Milgrom (1998) has recently reconsidered small groups in the context of MOND and finds that the statistically averaged mass-to-light ratio in groups is on the order of unity, removing the necessity of dark matter (Milgrom 1998). I previously considered 20 X-ray emitting clusters (Sanders 1994, Paper 1), and found that, for these objects, the mass predicted by MOND from the observed temperature of the hot gas is consistent with the inferred mass of hot gas. Moreover, MOND predicts the observed gas mass-temperature relation for clusters ( $M_g \propto T^2$ ) which is, in effect, the high mass continuation of the Faber-Jackson relation for elliptical galaxies.

This original sample of X-ray emitting clusters was from the early analysis of *Einstein* satellite data by Jones & Forman (1984) with temperature determinations by David et al. (1993). Recently, *Einstein* data for a much larger sample of 207 clusters has been compiled by White, Jones & Forman (1997) who are interested primarily in the properties of clusters with cooling flows. The purpose of the present note is to consider this larger sample in the context of MOND. I demonstrate below that, with modi-

fied dynamics, the observed temperatures (or velocity dispersions) of the clusters are consistent with the mass of hot gas estimated to be present within 1 to 3 Mpc of the cluster center.

## 2. The sample: gas mass and temperature determinations

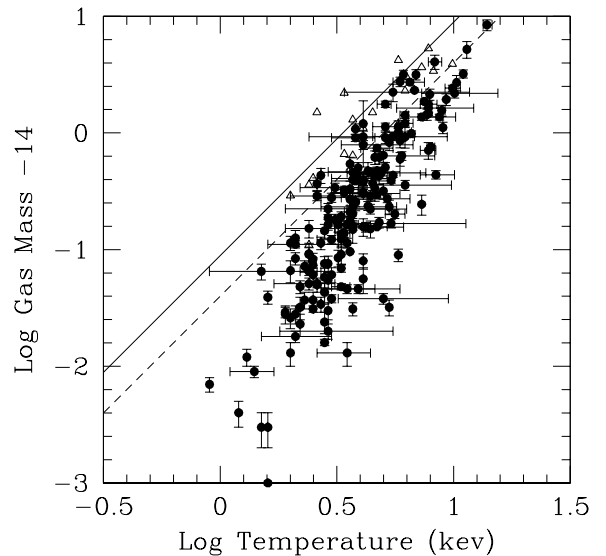
In their original sample, Jones & Forman estimated the gas mass inside 3 Mpc by fitting “ $\beta$  models” to the observed azimuthally averaged X-ray intensity distribution. The  $\beta$  model, which in most cases provides an adequate description of the mean run of X-ray intensity with projected radius, is characterized by a gas density distribution of the form

$$\rho = \rho_o \left[ 1 + \left( \frac{r}{r_c} \right)^2 \right]^{-1.5\beta} \quad (1)$$

where the core radius,  $r_c$ , the central density,  $\rho_o$ , and the number,  $\beta$  are parameters of the fit (see Sarazin 1988). The  $\beta$  parameter varies from cluster to cluster but has a typical value on the order of 2/3 which implies that the gas mass generally increases linearly with radius. Obviously this must steepen at some radius if the total gas mass is to converge, and it is clear that assumptions about the outer radius are critical in assigning a gas mass to a given cluster (so long as  $\beta \leq 1$ ). Jones & Forman estimated the gas mass by extrapolating this model fit out to 3 Mpc even though the actual parameters are usually determined by fitting over a smaller region.

In their new compilation, White et al. (1997) use a different method to estimate the gas mass— the “deprojection” method invented by Fabian et al. (1981). Here one assumes, in addition to perfect spherical symmetry, a model for the gravitational potential of the cluster— typically an isothermal sphere with a core radius and a velocity dispersion chosen to match that of the observed velocity dispersion of the galaxies or the mean X-ray temperature. One then assumes that the gas sits in hydrostatic equilibrium in this model potential and adjusts the gas density and temperature distribution in such a way as to reproduce the deprojected X-ray emissivity distribution.

The estimated mass of hot gas within a specific volume is fairly independent of the method used (i.e., the  $\beta$  model fitting or the deprojection method). This is because the derived gas mass depends primarily upon the X-ray luminosity and the volume of the emission region (see White et al. 1994, White & Fabian 1995). However, with the deprojection method, the gas mass is only determined within some radius,  $r_{out}$ , where the X-ray emission is lost in the background. This cut-off radius varies from about 0.1 Mpc for the low-luminosity, cooler clusters to 2 Mpc for the high-luminosity hot clusters. Therefore, the values of the gas mass tabulated by White et al. involve no uncertain extrapolation, but it is evident that some fraction of the gas mass is missing— a fraction which is



**Fig. 1.** A log-log plot of the mass of hot gas in units of  $10^{14} M_{\odot}$  vs. gas temperature in keV for 197 X-ray emitting clusters from the tabulation by White et al. 1997. The gas mass is that determined by the deprojection method inside a cutoff radius,  $r_{out}$ , which varies from cluster to cluster. The temperature is the “reference temperature” as defined by White et al. The error bars are shown where given. The MOND prediction (eq. 3,  $\beta = 2/3$ ) is shown by the solid line and the extrapolated Faber-Jackson relation by the dashed line.

probably greater in those clusters with smaller  $r_{out}$ , i.e., the cooler clusters.

## 3. Results

Taking the numbers directly from Table 3 of White et al. (1997), who assume a Hubble parameter of 50 km/s-Mpc, the gas mass is plotted against temperature in Fig. 1. There are 197 clusters plotted here. The mass of gas is that determined from the deprojection method inside radius  $r_{out}$  (column *viii*) and the temperature is the “reference” temperature defined from analysis of the X-ray spectrum for all of those clusters for which this is available (column *vi* in parenthesis). This reference temperature rather than the emission-weighted fitted temperature was used because this is comparable to that used in Paper 1 (the appearance of the plot is similar if the fitted temperature is used instead). The error bars in mass and temperature are shown for those cases where errors are given. The open triangles are data for the 20 clusters from Paper 1, i.e., gas mass estimates from extrapolated  $\beta$  models (Jones & Forman 1984).

These results are compared in Fig. 1 to the predictions of modified dynamics. In the low acceleration limit of MOND, the spherically symmetric equation of hydrostatic equilibrium (Milgrom 1984) may be immediately solved for the mass enclosed within radius  $r$ :

$$M_r = \frac{1}{G a_o} \left( \frac{k T_r}{\mu} \right)^2 \left[ \frac{d \ln \rho}{dr} + \frac{d \ln T}{dr} \right]_r^2 \quad (2)$$

where  $k$  is the Boltzmann constant;  $\mu$  is the mean molecular weight;  $a_o$  is the MOND acceleration parameter found to be  $0.8 \times 10^{-8} h_{50} \text{ cm/s}^2$  from galaxy rotation curves (Begeman et al. 1991); and the temperature and logarithmic gradients are given at radius  $r$ . For a  $\beta$  model, assumed to be isothermal, this may be written as

$$\left( \frac{M}{10^{14} M_\odot} \right) = 0.2 \beta^2 T^2 h_{50}^{-1} \quad (3)$$

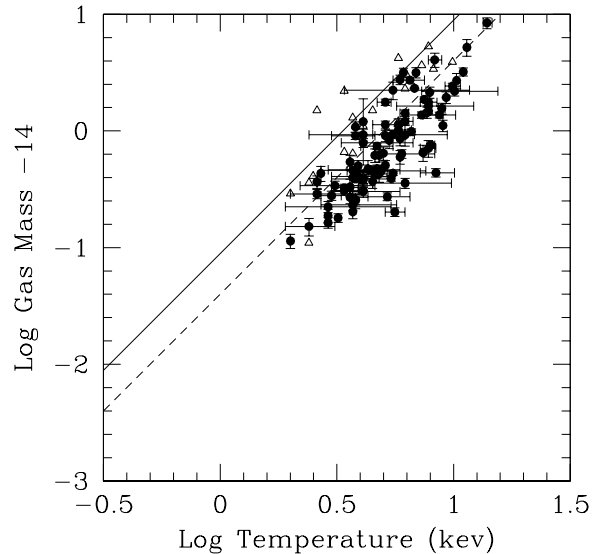
with the temperature in keV (Sanders 1994). This mass-temperature relation is shown by the solid line in Fig. 1 where  $\beta \approx 2/3$  is assumed.

The high mass extrapolation of the the observed Faber-Jackson relation for elliptical galaxies (Faber & Jackson 1976), as calibrated by Fukugita & Turner (1991) scaled to  $h_{50} = 1$ , is shown by the dashed line in Fig. 1, where the mass-to-light ratio for elliptical galaxies is assumed to be 7.5. This is shown here as well because, in the context of MOND, the gas mass-temperature relation for clusters is the high mass continuation of the Faber-Jackson relation for ellipticals, apart from the anisotropy factor (Milgrom 1984) and the uncertainty of the mass-to-light ratio (Paper 1).

We see in Fig. 1 that the points from the new enlarged cluster sample lie, except for the high temperature end, generally below the MOND prediction for isothermal spheres and the extrapolated Faber-Jackson relation for ellipticals and define a steeper relation (a linear least-square fit gives a slope of 2.5). The new mass estimates also lie below the those considered in Paper 1 which is not surprising because the estimates there were based upon extrapolations out to 3 Mpc. However, as noted above, the deprojection analysis misses a fraction of the gas which is probably greater in those clusters with smaller  $r_{out}$ — the lower temperature clusters.

There are two ways to correct for this effect. The fairest way is to consider only those systems in which  $r_{out}$  is larger than some specified value because this will eliminate clusters with the largest fraction of missing gas. This is shown in Fig. 2 where only clusters with  $r_{out} > 0.75$  Mpc (93 systems) are plotted. The agreement with the extrapolated Faber-Jackson law is evident, but the masses are typically a factor of two or three below the mass predicted by MOND for isothermal spheres with  $\beta = 2/3$ .

A second method of correcting for missing gas, less fair but more consistent with the extrapolated  $\beta$  models, is to multiply the gas masses given by White et al. by a factor of  $3/r_{out}$  for each cluster. Here one assumes that



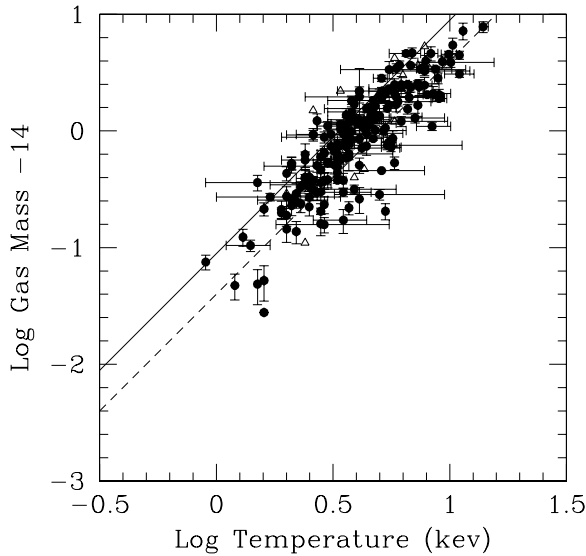
**Fig. 2.** Gas mass vs. temperature as in Fig. 1, but only for those cases (93 clusters) in which the cutoff radius for gas mass determination,  $r_{out}$ , exceeds 0.75 Mpc.

the gas mass scales as radius ( $\beta \approx 2/3$ ) at least out to 3 Mpc for all clusters. The results of this are shown in Fig. 3 where correlation between gas mass and temperature closely agrees in form and amplitude with the MOND prediction of  $M \propto T^2$ .

#### 4. Comments

Considering the errors in temperature and in the estimated gas mass, the scatter in the observed gas mass-temperature relation is easily understandable. To this should be added the effect of the several idealized assumptions which are certainly not realized in every case: spherical symmetry, hydrostatic equilibrium, isothermality, and thermal equilibrium between electrons and positive ions. Indeed, it is evident from eq. 2 that a pure  $\beta$  model cannot be realized for an isothermal sphere in the low acceleration limit of modified dynamics— the logarithmic density gradient must always steepen with radius. Moreover, the contribution of the stellar mass in galaxies to the total accounting of detectable matter cannot be neglected in all cases (particularly in the lower temperature clusters). Given these caveats, the approximate agreement of the extrapolated gas mass with predictions of eq. 3 (Figs. 2 & 3) demonstrates that MOND can account for the overall virial discrepancy in clusters within 1 to 3 Mpc.

However, there are actually two discrepancies in clusters of galaxies: that between the virial mass and the luminous mass and that between the lensing mass and the luminous mass. The discrepancy implied by weak lensing



**Fig. 3.** Gas mass vs. temperature as in Fig. 1 but the gas mass for all systems is scaled up by a factor of  $3/r_{out}$ . The assumption here is that the gas mass distribution is described by a  $\beta$  model with  $\beta = 2/3$  which continues out to 3 Mpc for every cluster. This is a considerable extrapolation in several cases (a factor of 30) and probably overestimates the total gas mass for some systems.

due to clusters of galaxies (the slight systematic distortion of the images of background galaxies) is generally consistent with the virial discrepancy to within a factor of two or three (Wu & Fang 1997, Allen 1998). Therefore any relativistic extension of MOND which preserves the relation between the weak field force and the deflection of light (i.e.,  $\theta = 2/c^2 \int g_{\perp} dl$ ) will also account for the lensing discrepancy (an example of such a theory is given in Sanders, 1997).

Some clusters also act as strong lenses; i.e., multiple images of background sources are formed by the central regions of the clusters. The critical surface density required for strong lensing is

$$\Sigma_c = \frac{1}{4\pi} \frac{cH_o}{G} F(z_l, z_s) \quad (4)$$

where  $F$  is a dimensionless function of the lens and source redshifts which depends upon the cosmological model (Blandford & Narayan 1992); typically for clusters and background sources at cosmological distances  $F \approx 10$ . Modified dynamics applies in the limit of low accelerations or, equivalently, at surface densities below a value of  $\Sigma_M \approx a_o/G$  (Milgrom 1983a). Since, observationally it is found that  $a_o \approx cH_o/6$ , this implies that  $\Sigma_c \approx 6\Sigma_M$ ; that is to say, the critical surface density for strong lensing is always greater than the upper limit for MOND phe-

nomenology. *Strong lensing always occurs in the Newtonian regime.* Strong lensing observed in clusters typically requires a total projected mass in the inner 100-200 kpc in excess of  $10^{14} M_{\odot}$  which is evidently not present in the form of hot gas. Hitherto undetected matter does seem to be necessary in the cores of rich clusters which exhibit strong lensing, even with modified dynamics (see also Milgrom 1996).

This may be taken as a failure or as a prediction. That the tally of ordinary baryonic matter may not yet be complete is suggested by the observations, in several clusters, of diffuse star light (Ferguson et al. 1998) and ultraviolet emission apparently from warm clouds (Mittaz et al. 1998). Moreover, many X-ray clusters show evidence for cooling flows at some level (Sarazin 1988). The gas cools, flows inward and disappears into some, as yet, undetectable form. The mass deposition rates are generally too low to be dynamically significant at the present epoch, but this may not have always been the case.

In summary, MOND goes a long way in resolving the virial discrepancy in clusters. It cannot resolve the strong lensing discrepancy in those clusters where this phenomenon is observed, but this leads to the prediction that more baryonic matter is present and possibly detectable in the cores of rich clusters.

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## References

- Allen, S.W. 1998, MNRAS, 296, 392
- Begeman, K.G., Broeils, A.H. & Sanders, R.H. 1991, MNRAS, 249, 523
- Blandford, R.D., Narayan, R. 1992, ARA&A, 30, 311
- David, L.P., Arnaud, K.A., Forman, W., Jones, C. 1990, ApJ, 356, 32
- David, L.P., Sylz, A., Jones, C., Forman, W., Vrtilek, S.D. 1993, ApJ, 412, 479
- Faber, S.M., Jackson, R.E. 1976, ApJ, 204, 668
- Fabian, A.C., Hu, E.M., Dowie, L.L., Grindley, J. 1981, ApJ, 248, 47
- Ferguson, H.C., Tanvir, N.R., von Hippel, T. Nature, 391, 461
- Fukugita, M., Turner, E.L. 1991, MNRAS, 253, 99
- Jones, C. Forman, W. 1984, ApJ, 276, 38
- McGaugh, S.S., de Blok, W.J.G. 1998a, ApJ, 499, 66
- McGaugh, S.S., de Blok, W.J.G. 1998b, submitted to ApJ
- Milgrom, M. 1983a,b,c ApJ, 270, 365
- Milgrom, M. 1984, ApJ, 287, 571
- Milgrom, M. 1996, astro-ph/9601080
- Milgrom, M. 1998, ApJ, 496, 89
- Mittaz, J.P.D., Lieu, R., Lockman, F.J. 1998, ApJ, 498, L17
- Navarro, J.F., Frank, C.S., White, S.D.M. 1996, ApJ, 462, 563
- Sanders, R.H. 1994, A&A, 284, L31 (Paper 1)
- Sanders, R.H. 1996 ApJ, 473, 117
- Sanders, R.H. 1997 ApJ, 480, 492
- Sanders, R.H., Verheijen M.A.W. 1998 ApJ (in press)
- Sarazin, C.L. 1988, *X-ray Emissions from Clusters of Galaxies*, Cambridge University Press
- White, D.A., Fabian, A.C. 1995, MNRAS, 273, 72

- White, D.A. et al. 1994, MNRAS, 269, 589  
White, D.A., Jones, C., Forman, W. 1997, MNRAS, 292, 419  
Wu, X., Fang, L. 1997, ApJ, 483, 62  
Zwicky, F. 1933, *Helv.Phys.Acta*, 6, 110